A Fine-Grained Classification of Subquadratic Patterns for Subgraph Listing and Friends

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Saarland University and Max Planck Institute for Informatics

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SIC Saarland Informatics Campus

MAX PLANCK INSTITUTE FOR INFORMATICS

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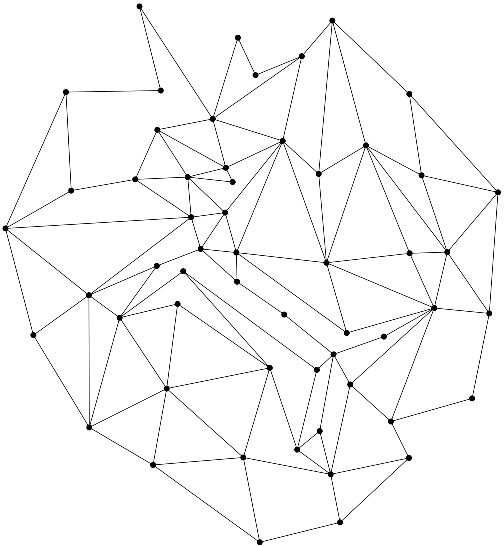
Example



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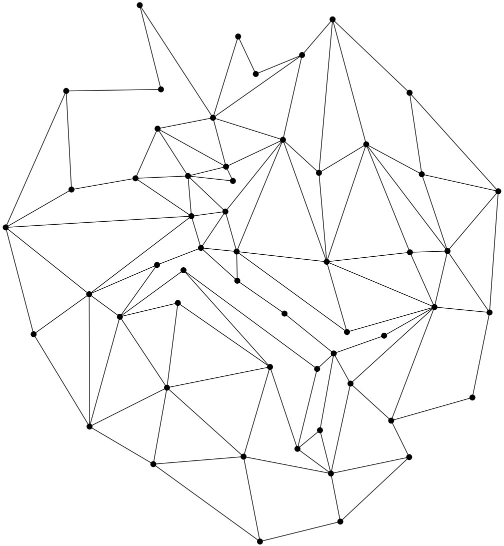

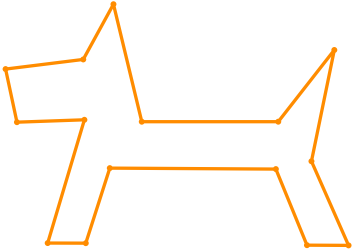

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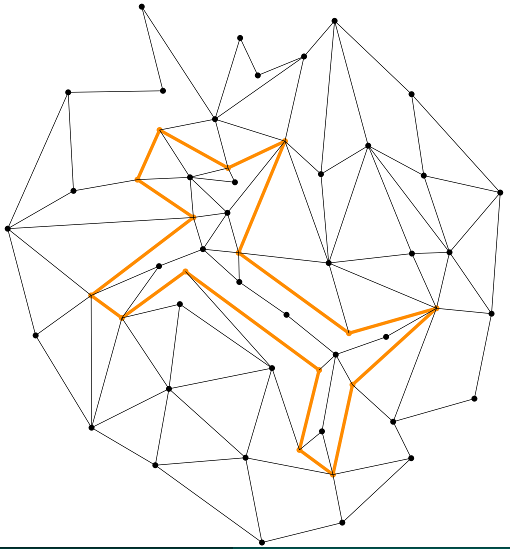

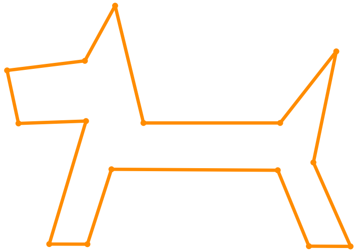

Problem Variants

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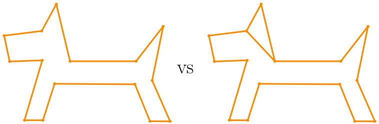
- Non-induced



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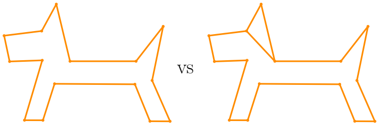

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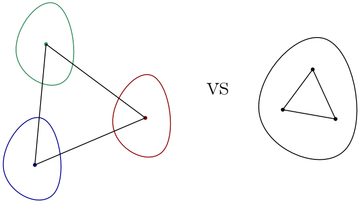

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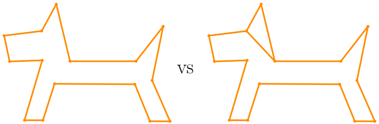

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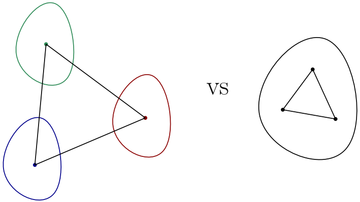

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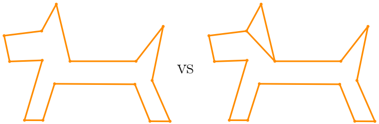

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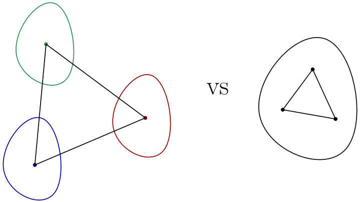

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Problem Definition

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Fix pattern H. **Min-weight (colored) H-subgraph isomorphism problem:**

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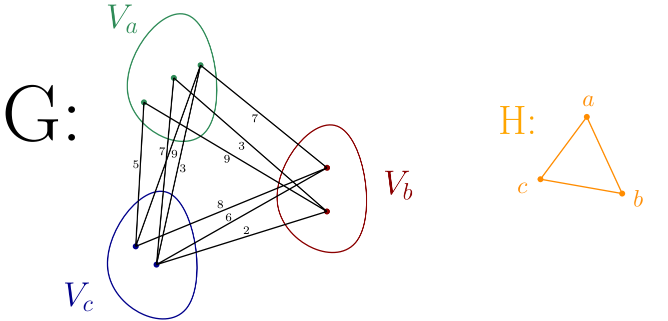

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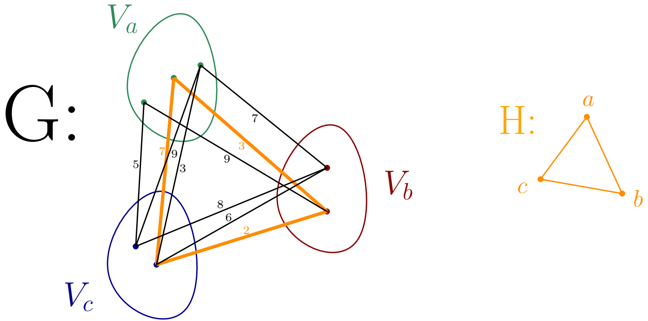

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- [Marx10]: $m^{\mathcal{O}(\mathrm{tw}(H))}$ algorithms vs $m^{\Omega(\mathrm{tw}(H)/\log \mathrm{tw}(H))}$ lower bounds gap in general.
  $\hookrightarrow$ Our goal: characterize patterns of low time complexity.

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Folklore: $\mathcal{O}(m)$

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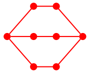

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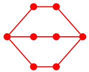

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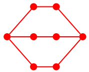

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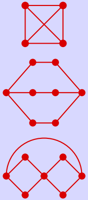

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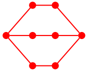

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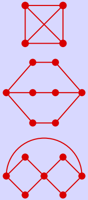

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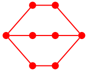

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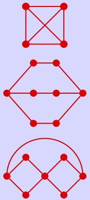

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Everything Else

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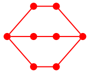

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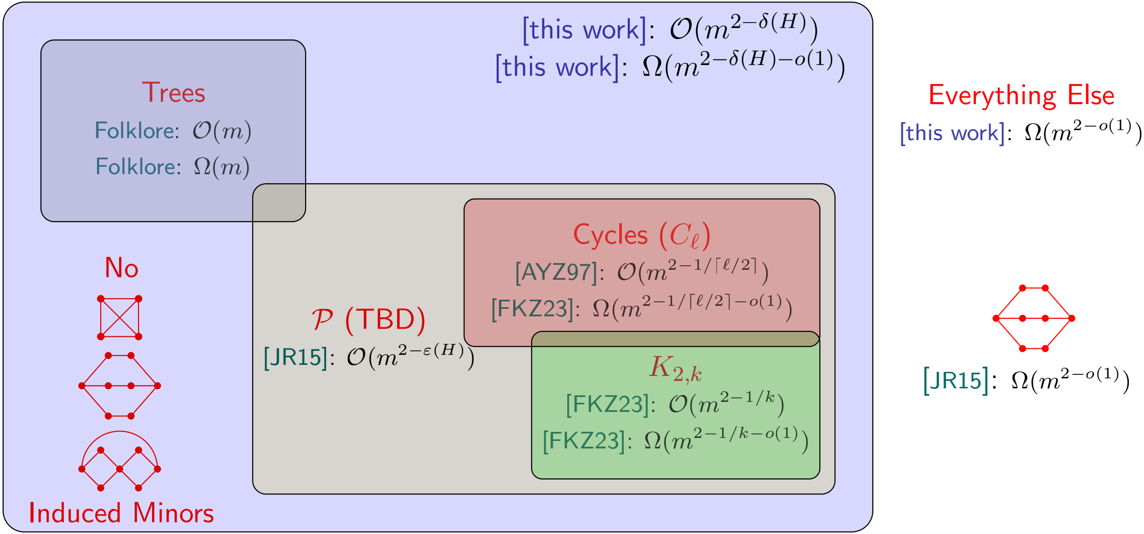
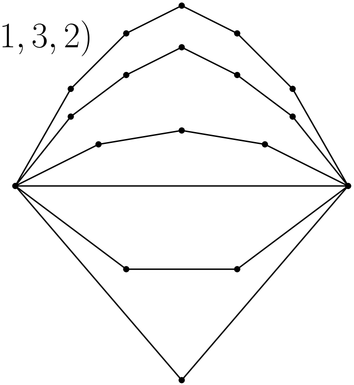


Definition (Parallel Path Graph)

A *Parallel Path Graph* $P(\ell_1, \ell_2, \ldots, \ell_k)$ consists of two specified vertices connected by k internally disjoint paths of lengths ℓ_1, ℓ_2, …, ℓ_k.

$P(6, 6, 4, 1, 3, 2)$



Building Blocks of Subquadratic Patterns

Definition (Subquadratic Building Blocks) [JR15]

Let $\mathcal{P}$ be the family of patterns consisting of:

- A single edge $(P(1))$;
- A triangle $(P(2,1))$;
- $P(\alpha, \beta, \underbrace{2, 2, \ldots, 2}_{\gamma \text{ times}})$ for all $\alpha \geq \beta \geq 2$, $\gamma \geq 0$ (denote by $P(\alpha, \beta, \gamma \times 2)$).

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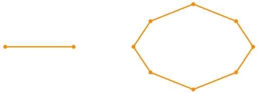

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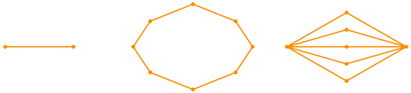

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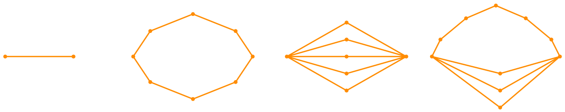

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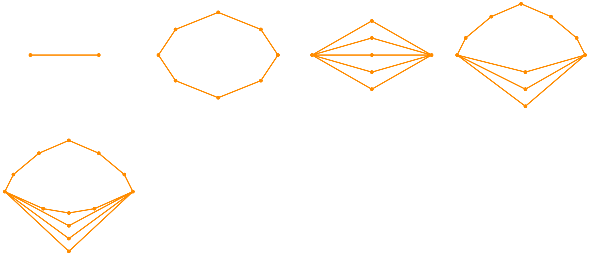

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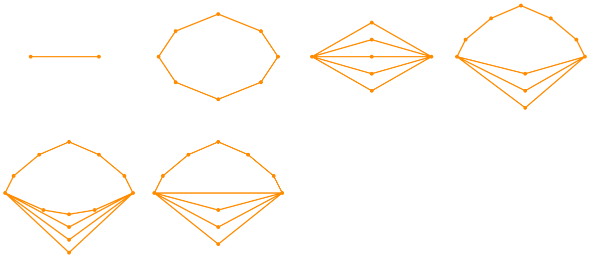

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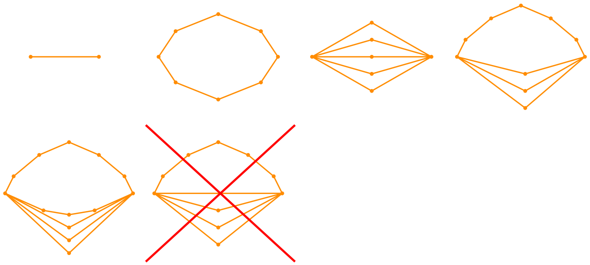

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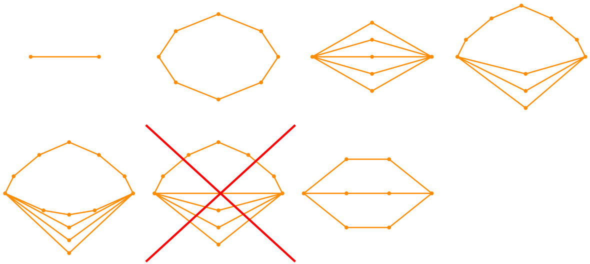

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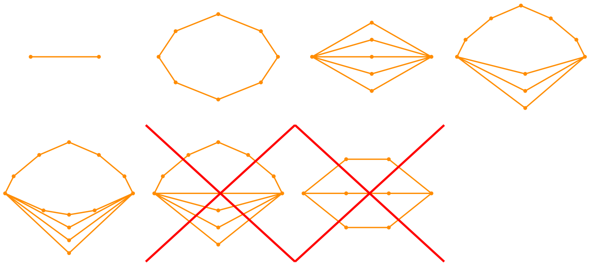

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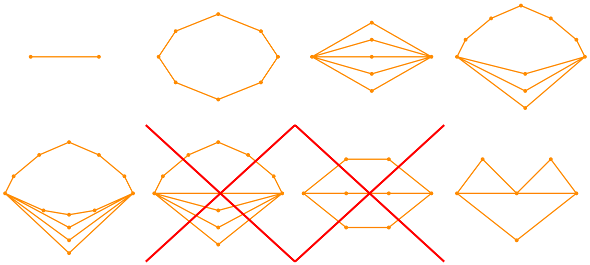

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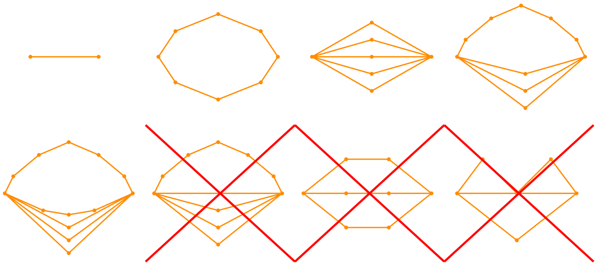

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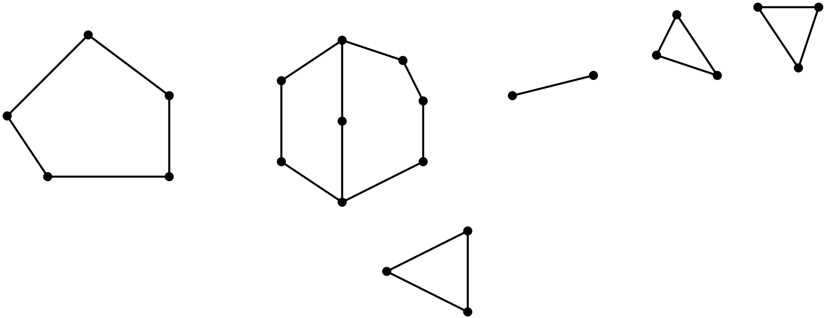

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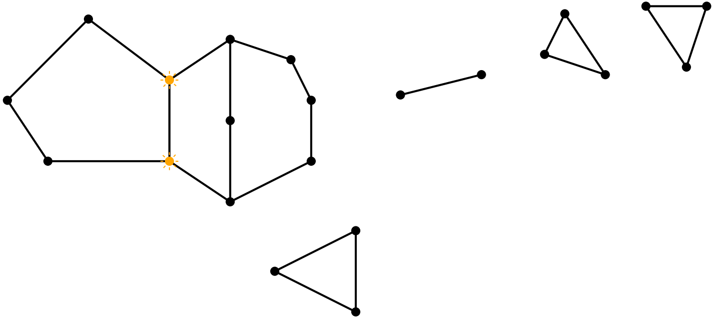

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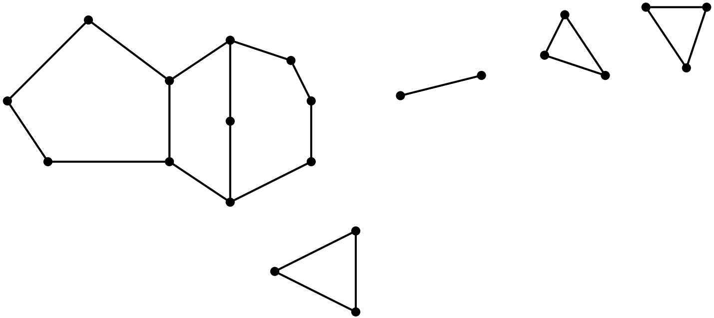

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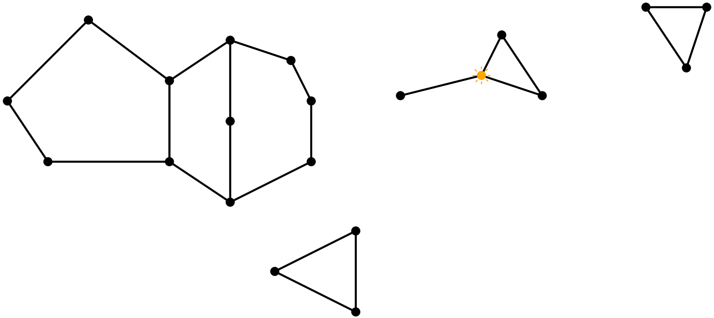

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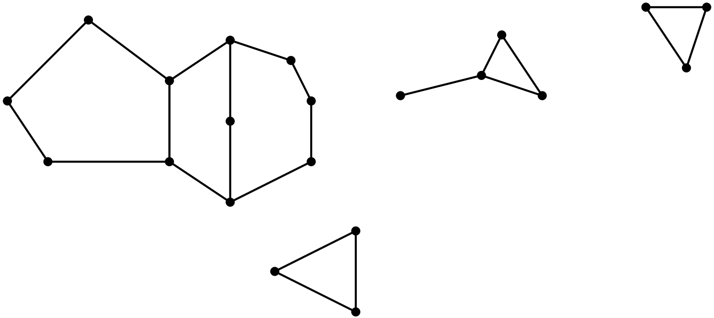

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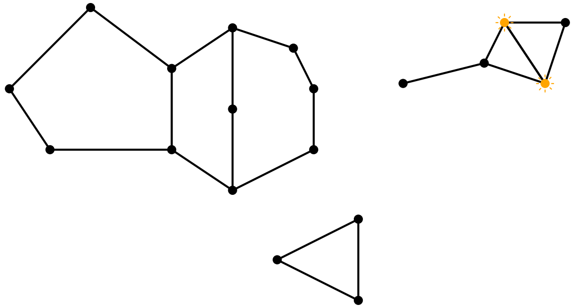

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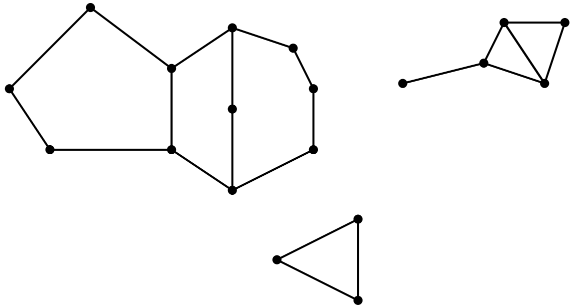

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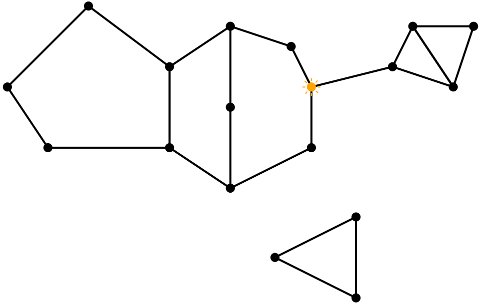

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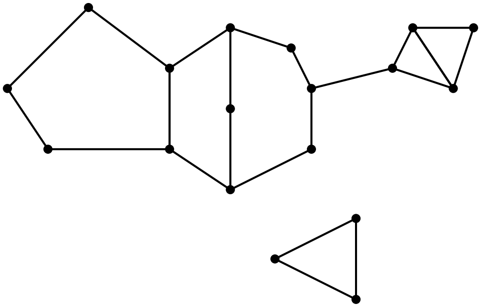

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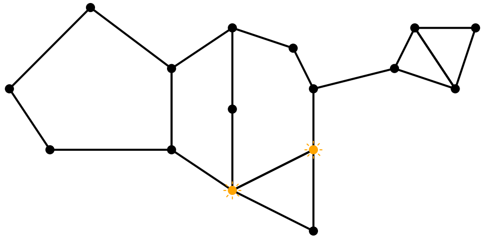

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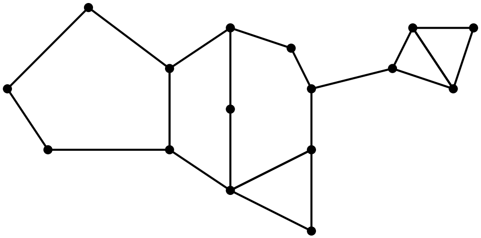

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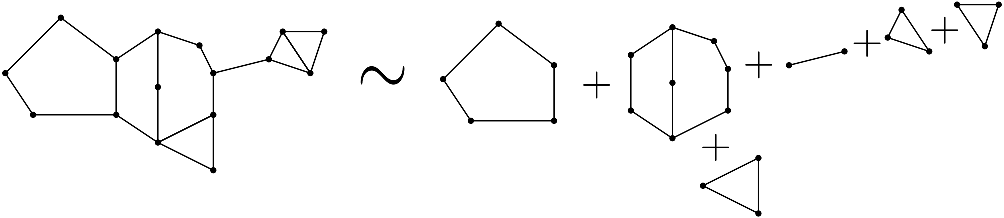

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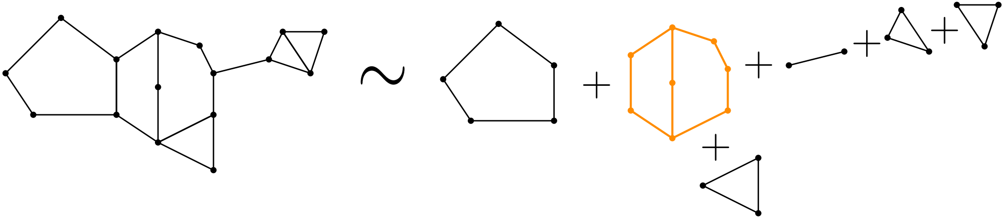

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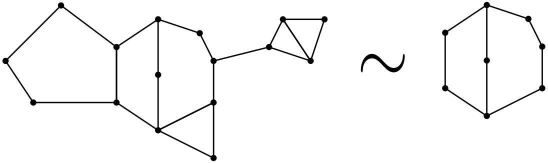

Main Result: Quantitative Version

Main Technical Theorem

Min-weight $P(\alpha, \beta, \gamma \times 2)$-subgraph isomorphism is solvable in time $O(m^{2-1/f(\alpha,\beta,\gamma)})$ and not solvable in time $O(m^{2-1/f(\alpha,\beta,\gamma)-\varepsilon})$ for any $\varepsilon > 0$ under fine-grained complexity assumptions, where

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$$f(\alpha,\beta,\gamma) = \begin{cases} 2\beta\gamma + \frac{\alpha\beta}{2} - \frac{\beta^2}{2} + \frac{\beta}{2} - \frac{\alpha}{2} - 2\gamma + 2, & \text{if } \alpha+\beta \text{ is even, } \alpha > \beta, \text{ and } \beta < \gamma + 2; \\ 2\beta\gamma + \frac{\alpha\beta}{2} - \frac{\beta^2}{2} + \frac{3\beta}{2} - \frac{\alpha}{2} - 3\gamma, & \text{if } \alpha+\beta \text{ is even, } 3\beta < \alpha + 6\gamma + 8, \text{ and } (\alpha = \beta \text{ or } \beta \geq \gamma + 2); \\ 2\beta\gamma + \frac{\alpha\beta}{2} - \frac{\beta^2}{2} + 3\beta - \alpha - 6\gamma - 4, & \text{if } \alpha+\beta \text{ is even, } 2\beta \leq \alpha + 4\gamma + 6, \text{ and } 3\beta \geq \alpha + 6\gamma + 8; \\ 2\beta\gamma + \frac{\alpha\beta}{2} - \frac{\beta^2}{2} + \beta - \frac{\alpha}{2} - 2\gamma + \frac{3}{2}, & \text{if } \alpha+\beta \text{ is odd and } \beta < 2\gamma + 3; \\ 2\beta\gamma + \frac{\alpha\beta}{2} - \frac{\beta^2}{2} + 2\beta - \frac{\alpha}{2} - 4\gamma - \frac{3}{2}, & \text{if } \alpha+\beta \text{ is odd, } 2\beta \leq \alpha + 4\gamma + 6, \text{ and } \beta \geq 2\gamma + 3; \\ 2\gamma^2 + \alpha\gamma + \frac{\alpha^2}{8} + \frac{\alpha}{2}, & \text{if } \alpha = 0 \bmod 4 \text{ and } 2\beta > \alpha + 4\gamma + 6; \\ 2\gamma^2 + \alpha\gamma + \frac{\alpha^2}{8} + \frac{\alpha}{2} + \frac{3}{8}, & \text{if } \alpha \text{ is odd and } 2\beta > \alpha + 4\gamma + 6; \\ 2\gamma^2 + \alpha\gamma + \frac{\alpha^2}{8} + \frac{\alpha}{2} + \frac{1}{2}, & \text{if } \alpha = 2 \bmod 4 \text{ and } 2\beta > \alpha + 4\gamma + 6. \end{cases}$$

$\mathcal{P}^+$

[this work]: $\mathcal{O}(m^{2-\delta(H)})$

[this work]: $\Omega(m^{2-\delta(H)-o(1)})$

Trees

Folklore: $\mathcal{O}(m)$

Folklore: $\Omega(m)$

$\mathcal{P}$

[JR15]: $\mathcal{O}(m^{2-\varepsilon(H)})$

Cycles (C_ℓ)

[AYZ97]: $\mathcal{O}(m^{2-1/\lceil \ell/2 \rceil})$

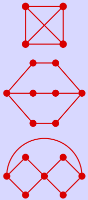
[FKZ23]: $\Omega(m^{2-1/\lceil \ell/2 \rceil - o(1)})$

$K_{2,k}$

[FKZ23]: $\mathcal{O}(m^{2-1/k})$

[FKZ23]: $\Omega(m^{2-1/k-o(1)})$

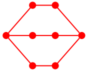
No



Induced Minors

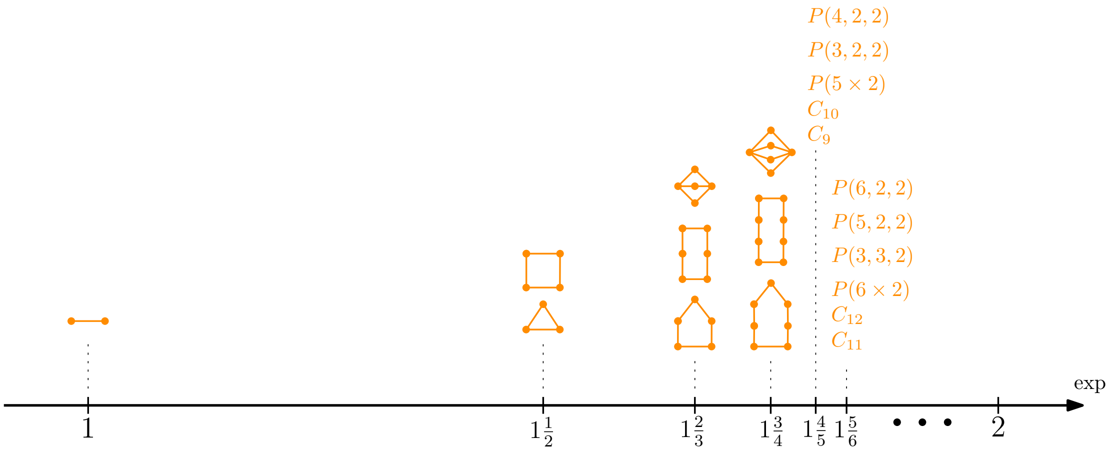
Everything Else

[this work]: $\Omega(m^{2-o(1)})$



[JR15]: $\Omega(m^{2-o(1)})$

Exponent Examples



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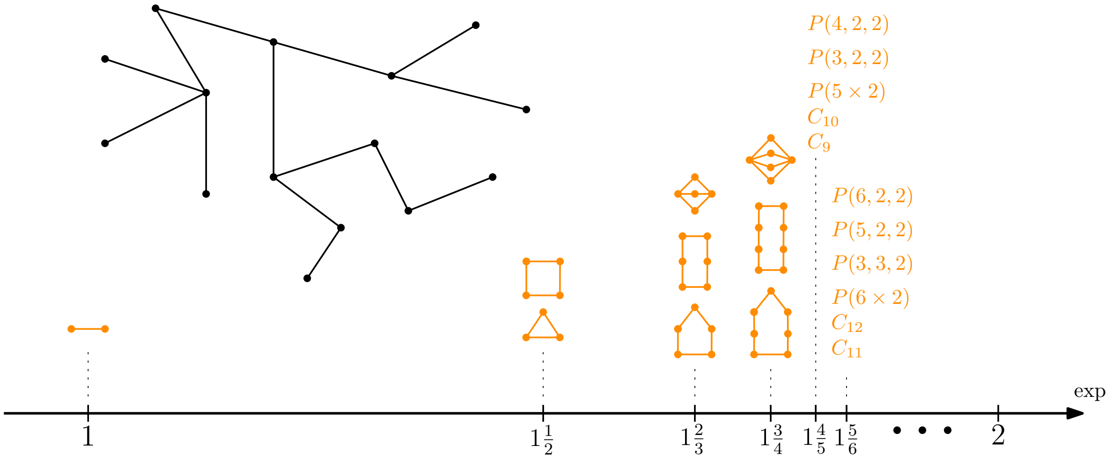

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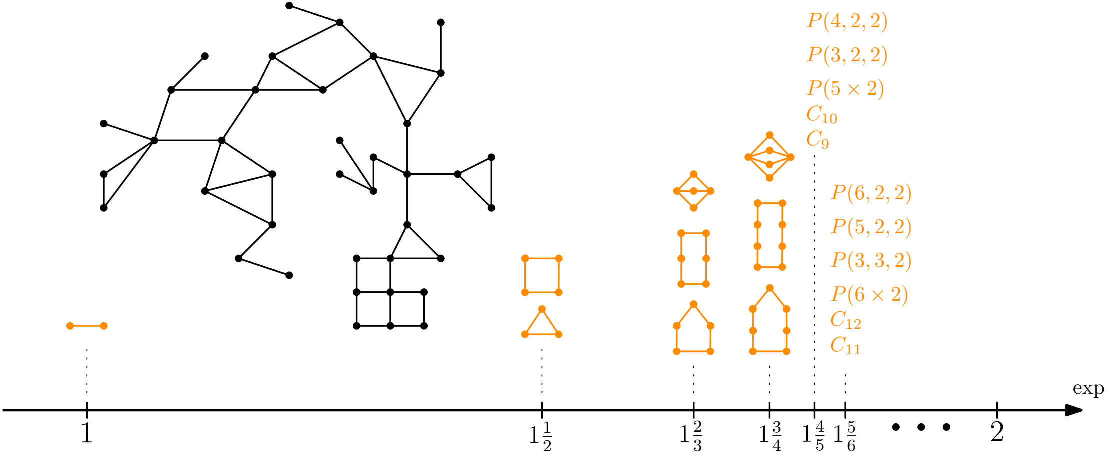

Summary and Open Problems

Main Theorem

There is an explicitly defined set of patterns $\mathcal{P}^+$ such that:

If $H \notin \mathcal{P}^+ \Rightarrow \Omega(m^{2-o(1)})$ conditional lower bound for min-weight colored H-subgraph isomorphism.

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Paper PDF

Long talk recording

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