Bounded Edit Distance

Optimal Static and Dynamic Algorithms for Small Integer Weights

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SIC Saarland Informatics Campus

MAX PLANCK INSTITUTE FOR INFORMATICS

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Weighted Edit Distance

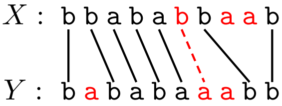
Weighted Edit Distance $\mathsf{ed}^w(X,Y)$ — $w\colon (\Sigma\cup\{\varepsilon\})\times(\Sigma\cup\{\varepsilon\})\to\mathbb{R}_{\ge 0}$

The minimum cost of transforming X into Y by editing individual characters, where:

- inserting b costs $w(\varepsilon, b)$;
- deleting a costs $w(a, \varepsilon)$;
- substituting a for b costs $w(a, b)$.

w :

	ε	a	b
ε	0	1	4
a	1	0	2
b	3	2	0



$\mathsf{ed}^w(X,Y) = 6$

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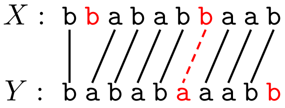

$\mathrm{ed}(X,Y) = 3$

Static State of the Art

Notation: $\boldsymbol{n = |X| + |Y|}$

Reference	*Time*	*Weights*
[Vin68,NW70,Sel74,WF74]	$\mathcal{O}(n^2)$	any

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Maintain strings $X, Y \in \Sigma^{\leq n}$ subjects to updates (character edits) and report $\mathsf{ed}^w(X, Y)$ after each update.

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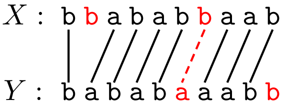

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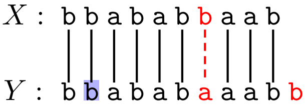

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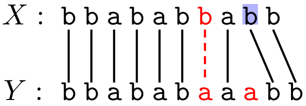

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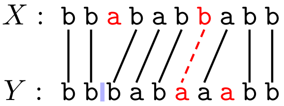

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