Bounded Weighted Edit Distance

Dynamic Algorithms and Matching Lower Bounds

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Weighted Edit Distance

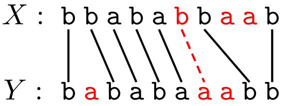
Weighted Edit Distance $\mathsf{ed}^w(X, Y)$ $\quad w\colon (\Sigma \cup \{\varepsilon\}) \times (\Sigma \cup \{\varepsilon\}) \to \mathbb{R}_{\geq 0}$

The minimum cost of transforming X into Y by editing individual characters, where:

- inserting b costs $w(\varepsilon \mapsto b)$;
- deleting a costs $w(a \mapsto \varepsilon)$;
- substituting a for b costs $w(a \mapsto b)$.

w :

	ε	a	b
ε	0	1	4
a	1	0	2
b	3	2	0



$\mathsf{ed}^w(X, Y) = 6$

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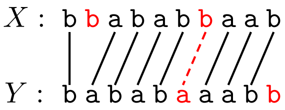

$\mathrm{ed}(X, Y) = 3$

Static State of the Art

Notation: $n = |X| + |Y|$

Reference	*Time*	*Weights*
[Vin68,NW70,Sel74,WF74]	$\mathcal{O}(n^2)$	any

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[CKW23]	$\widetilde{\mathcal{O}}(n + k^3)$	$\mathbb{R}_{\geq 1}$	
[CKW23]	$\widetilde{\mathcal{O}}(n + \sqrt{nk^3})$	$\mathbb{R}_{\geq 1}$	under APSPH, $\sqrt{n} \leq k \leq n$

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Problem

Maintain strings $X, Y \in \Sigma^{\leq n}$ subjects to updates (character edits) and report $\mathsf{ed}^w(X, Y)$ after each update.

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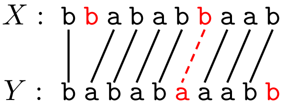

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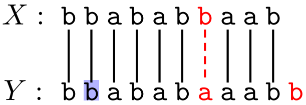

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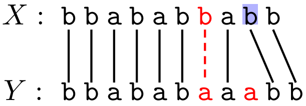

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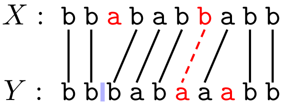

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[this work]	$\widetilde{\mathcal{O}}(k^2)^\star$	$\mathbb{R}_{\geq 1}$	

The Asterisk

Theorem [CKW23]

Assuming the APSP Hypothesis, there is no $\mathcal{O}(\sqrt{nk^{3-\varepsilon}})$-time algorithm for computing $\mathsf{ed}^w(X,Y)$ for instances satisfying $\sqrt{n} \le k \le n$.

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Fix k. Maintain $\mathrm{ed}^w_{\le k}(X,Y) = \begin{cases} \mathrm{ed}^w(X,Y) & \text{if } \mathrm{ed}^w(X,Y) \le k, \\ \infty & \text{otherwise.} \end{cases}$

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For every $\gamma \in [0,1]$, there is a dynamic algorithm with $\widetilde{\mathcal{O}}(nk^{\gamma})$ preprocessing and $\widetilde{\mathcal{O}}(k^{3-\gamma})$ update time that dynamically maintains $\mathrm{ed}^{w}_{\leq k}(X,Y)$.

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For $\gamma \in [0.5, 1)$, the update time of Theorem 3 cannot be improved by any polynomial factor assuming the APSP Hypothesis.

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Theorem 3 (Extended Version) [this work]

For every $\gamma \in [0,1]$, there is a dynamic algorithm that maintains a string X and after $\widetilde{\mathcal{O}}(nk^{\gamma})$-time preprocessing supports the following operations:

- Apply a character edit, substring deletion, or copy-paste to X in $\widetilde{\mathcal{O}}(k^2)$ time.
- Given query-access to testing equality of substrings of X and Y, compute $\mathrm{ed}^{w}_{\leq k}(X,Y)$ in $\widetilde{\mathcal{O}}(k^{3-\gamma})$ time.

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  - Matching conditional lower bound for $\gamma \in [0.5,1)$.
  - New $\Omega(\sqrt{nk^{3-o(1)}})$ conditional lower bound for $k \in [\sqrt{n}, n]$ for *statically* computing $\mathsf{ed}^w(X,Y)$ for strings satisfying $\mathsf{ed}(X,Y) = \mathcal{O}(1)$.

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Thank you!